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Subsumption and Incompatibility between Principles in Ranking-based Argumentation

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***Abstract*— Ranking-based semantics are a way of assessing the acceptability of arguments in an abstract argumentation framework, by providing a ranking on arguments. This paper aims at going towards a generalization of the construction of such semantics, by investigating subsumption and incompatibility cases that may arise when principles that may enter into their composition are combined.**

I. Introduction

Abstract argumentation is a reasoning model based on interacting arguments. Arguments and their interactions can be represented as a graph, as proposed by Dung [15]. Given such a framework, *semantics* have been defined by Dung and by others, as a way of determining which subsets of arguments (called extensions) can be collectively considered as acceptable. An overview of such extension-based semantics can be found in [6].

For reasoning or computational purposes, semantics may have to be encoded in logic as first done in [9] (later work include [17], [4], [22], [16], [5], [20]). A systematic approach for this encoding was proposed for extension-based semantics in [10], and a software, SESAME, that allows a user to get the logical encoding for a semantics of her own, has been developed [11]. This approach encodes a number of semantic principles, which can be combined, so as to obtain many of the existing semantics, or in order to create original ones.

Ranking-based semantics is a more recent family of argumentation semantics. When applied to an argument graph, such semantics output one or several preorders (called *rankings*) on the set of arguments. Hence, an argument can be said to be more acceptable than another, instead of being just an element of a collectively acceptable set. A comparative study of such semantics that output a single ranking, can be found in [13].

This paper aims at providing elements towards a generalization of the construction of ranking-based semantics. A number of *principles* which can be used for this construction have already been highlighted. Someone may want to combine some of them in order to define her own semantics. It may be useful however to indicate the user whether a certain principle may in fact not be needed, because following from other principles in the combination (case of *subsumption* between principles). Moreover, a certain combination may lead to *incompatibilities*, in the sense that a principle may give as a result that an argument a is at least as acceptable as an argument b, whereas another principle of interest to the user makes b to be strictly more acceptable than a. A special case of incompatibilities is when a principle is incompatible with itself (in this case, we say that this principle is *floundering*), and hence, incompatible with any other principle.

This paper investigates subsumption and different notions of incompatibility between principles, and provides results as to when such situations may arise.

The organization of the paper is as follows. Section II gives the relevant background on abstract argumentation. Section III defines and illustrates what ranking-based semantics are. Principles that may enter into the construction of such semantics are presented in Section IV. Subsumption results regarding principles are shown in Section V, incompatibility definitions and results are presented in Section VI. Section VII concludes.

II. Abstract argumentation

The notion of an argument graph is due to Dung in [15][1].

Definition 1: An *argument graph* is a couple $\langle A, R\rangle$ such that A is a finite set and $R \subseteq A \times A$ is a binary relation over A.

The set of vertices A is viewed as a set of abstract *arguments* — the origin and the structure of these are unspecified. The edges R represent *attacks*: $(a,b) \in R$, also written aRb, means that a attacks b. A set of arguments S attacks an argument a if a is attacked by some element of S.

Notation 1: The following abbreviations [14] are useful:

$$R_1^-(a) \overset{def}{=} \{b \in A \mid bRa\}$$
$$R_2^+(a) \overset{def}{=} \{c \in A \mid \exists b \in A,\ cRb \ \&\ bRa\}$$

$R_1^-(a)$ denotes the set of (direct) attackers of a and $R_2^+(a)$ denotes the set of arguments that (directly) *defend* a (that is, the set of arguments which attack an attacker of a).
These notations generalize: For i odd and j even s.t. $i,j > 2$:

$$R_i^-(a) \overset{def}{=} \{b \in A : \exists c \in R_{i-1}^+(a),\ bRc\}$$
$$R_j^+(a) \overset{def}{=} \{b \in A : \exists c \in R_{j-1}^-(a),\ bRc\}$$

$R_i^-(a)$ denotes the set of *indirect attackers* of a at level i, $R_j^+(a)$ the set of *indirect defenders* of a at level j (see [14]).

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[1]Dung uses the term argumentation framework instead of argument graph.

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Example 1: Figure 1 shows an example of an argument graph $F = \langle A, R\rangle$, taken from [14]: $A = \{a,b,c,d,e\}$ and $R = \{(b,a),(b,c),(c,e),(e,d),(d,a),(a,e)\}$. The set of attackers of a is $R_1^-(a) = \{b,d\}$, the set of its defenders is $R_2^+(a) = \{e\}$. The set of indirect attackers of a at level 3 is $R_3^-(a) = \{a,c\}$, the set of its indirect defenders at level 4 is $R_4^+(a) = \{b,d\}$.

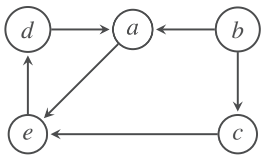

Fig. 1. An argument graph F

III. Ranking-based semantics

This section defines what a ranking-based semantics is. The definition is illustrated with two existing semantics.

*Definition 2 (**Ranking-based semantics**):* A *ranking-based semantics* σ associates to any argumentation framework $F = \langle A,R\rangle$ a ranking $\preceq_F^\sigma$ on A, where $\preceq_F^\sigma$ is a preorder (a reflexive and transitive relation) on A. $a \preceq_F^\sigma b$ means that b is at least as acceptable as a according to σ in F.

When there is no ambiguity on the argument graph that is considered, $\preceq^\sigma$ is used instead of $\preceq_F^\sigma$. In addition, when there is no ambiguity on the semantics, $\preceq$ is used instead of $\preceq^\sigma$.

Notation 2: $a \prec b$ abbreviates $a \preceq b \ \wedge \ b \not\preceq a$; it means that b is strictly more acceptable than a. $x \simeq y$ abbreviates $x \preceq y \ \wedge \ y \preceq x$ with the meaning that x is as acceptable as y.

The preorder may be defined by weighing arguments. This is the approach taken by Categorizer, a ranking-based semantics introduced in [12] (extended in [21]).

*Definition 3 (**Categorizer**):* For $F = \langle A,R\rangle$, $Cat : A \to]0,1]$ is defined as

$$Cat(a) = \begin{cases} 1 & \text{if } R_1^-(a) = \emptyset \\ \dfrac{1}{1 + \sum\limits_{c \in R_1^-(a)} Cat(c)} & \text{otherwise} \end{cases}$$

The ranking-based semantics *Categorizer* associates to any $F = \langle A,R\rangle$ a ranking $\preceq_F^{Cat}$ on A such that $\forall a,b \in A$, $a \preceq_F^{Cat} b$ iff $Cat(b) \geq Cat(a)$.

Stated otherwise, Categorizer assigns a value to each argument depending on the weight of its direct attackers.

Example 2: Consider the graph of Figure 1. The ranking that is given by Categorizer is as follows (the Categorizer value of each argument is indicated below the argument):

$$\begin{array}{ccccccccc} a & \prec^{Cat} & c & \prec^{Cat} & e & \prec^{Cat} & d & \prec^{Cat} & b \\ 0.38 & & 0.5 & & 0.53 & & 0.65 & & 1 \end{array}$$

Discussion-based semantics (Dbs) has been proposed in [1]. Arguments are compared by counting the number of direct attackers, and by calculating a score according to the level of the attackers of the argument at hand. If there is an equality up to a certain level, Dbs looks recursively into the attack paths.

Definition 4: Let $F = \langle A,R\rangle$. The *discussion count* of $a \in A$ is defined as $Dis(a) = \langle Dis_1(a), Dis_2(a), \ldots\rangle$ where, for $i \in \mathbb{N}$,

$$Dis_i(a) = \begin{cases} -|R_i^+(a)| & \text{if } i \text{ is even,} \\ |R_i^-(a)| & \text{if } i \text{ is odd.} \end{cases}$$

Discussion-based semantics is defined using the notion of lexicographical order.

Definition 5: A *lexicographical order* between two vectors of real numbers $V = \langle V_1, \ldots, V_n\rangle$ and $V' = \langle V'_1, \ldots, V'_n\rangle$, is defined as $V' \preceq_{lex} V$ iff $\exists i \leq n$ s.t. $V_i \geq V'_i$ and $\forall j < i$, $V_j = V'_j$.

*Definition 6 (**Dbs**):* The ranking-based semantics *Dbs* ascribes every $F = \langle A,R\rangle$ a ranking $\preceq_F^{Dbs}$ on A such that $\forall a,b \in A$, $a \preceq_F^{Dbs} b$ iff $Dis(b) \preceq_{lex} Dis(a)$.

Dbs assigns a preorder on arguments, but, contrariwise to Categorizer, it does not assign any value to arguments.

Example 3: Consider the graph of Figure 1. In order to rank the arguments, we must count the number of attackers for each argument until a difference shows up or the path is over.

step	*a*	*b*	*c*	*d*	*e*
1	2	0	1	1	2
2	−1	0	0	−2	−3

Here, the ranking given by the Discussion-based semantics is:

$$a \quad \prec^{Dbs} \quad e \quad \prec^{Dbs} \quad c \quad \prec^{Dbs} \quad d \quad \prec^{Dbs} \quad b$$

We can notice that this ranking on arguments differs from the one obtained with Categorizer: c is strictly more acceptable than e in the one, whereas it is e which is strictly more acceptable than c in the other.

Other ranking-based semantics include [19], [18], [13], [3]. A comparative study is provided in [14]. Ranking-based semantics that only assign a preorder on arguments, but no value at all to arguments (like Dbs), are called *pure* ranking-based semantics. It is on principles for the construction of such semantics that we focus in the sequel.

IV. Principles for ranking-based semantics

Existing ranking-based semantics that assign a numerical degree to every argument, have been analyzed according to an insightful series of axioms [2]. Some of these axioms are relevant for pure ranking-based semantics. Moreover, some of them can be turned into construction principles.

By ranking-based semantic *principles*, we actually mean properties taking the form:

$$\text{for all } \langle A,R\rangle, \text{ for all } a,b \in A, \text{ if } \ldots \text{ then } a \preceq b$$

where $\preceq$ is supposed to be reflexive and transitive (preorder).

Alternatively, a principle can be of the form:

$$\text{for all } \langle A,R\rangle, \text{ for all } a,b \in A, \text{ if } \ldots \text{ then } a \prec b$$

Not only is $\prec$ required to be transitive, it is also assumed that $x \prec z$ ensues from $x \prec y$ and $y \preceq z$ (from $x \preceq y$ and $y \prec z$, too).

Notation 3: $P_\alpha[a,b]$ denotes the ... part (*proviso*) of a principle α and $C_\alpha[a,b]$ denotes the other part (*conclusion*).

Various existing principles of interest are as follows.

(VP) Given any $\langle A,R \rangle$, for all $a,b \in A$, if $R_1^-(b) = \emptyset$ and $R_1^-(a) \neq \emptyset$ then $a \prec b$.

The idea behind **Void Precedence (VP)** (introduced in [19]) is that an argument, if not attacked, is strictly more acceptable than an argument that is attacked.

Notice that the proviso part $P_{\mathbf{VP}}[a,b]$ of **VP** is $R_1^-(b) = \emptyset$ and $R_1^-(a) \neq \emptyset$, and its conclusion $C_{\mathbf{VP}}[a,b]$ is $a \prec b$.

(SC) Given any $\langle A,R \rangle$, for all $a,b \in A$, if $a \in R_1^-(a)$ and $b \notin R_1^-(b)$ then $a \prec b$.

An argument that attacks itself (a *self-attacking* argument) is strictly less acceptable than an argument that does not attack itself; this is what **Self Contradiction (SC)** [19] indicates.

(EQ) Given any $\langle A,R \rangle$, for all $a,b \in A$ and for all $c \in R_1^-(a)$: if $|R_1^-(a)| = |R_1^-(b)|$ and $|\{x \in R_1^-(a) \mid x \simeq c\}| = |\{x \in R_1^-(b) \mid x \simeq c\}|$ then $a \simeq b$.

According to **Equivalence (EQ)** [2], if two arguments have the same number of attackers, and if the attackers of one of these two are as acceptable as the attackers of the other, then the two arguments are as acceptable as each other.

(CT) Given any $\langle A,R \rangle$, for all $a,b \in A$ where $R_1^-(a) = \{a_1,\ldots,a_n\}$ and $R_1^-(b) = \{b_1,\ldots,b_{n'}\}$:

if $n \geq n'$ and there exists a permutation ρ over $R_1^-(a)$ such that $\forall i \in [1,n']$ $b_i \preceq a_{\rho(i)}$

then $a \preceq b$

If an argument a has at least as many attackers as an argument b, and if the attackers of a are at least as acceptable as the attackers of b, then, according to **Counter-Transitivity (CT)** (introduced in [1]), b is at least as acceptable as a.

(SCT) Given any $\langle A,R \rangle$, for all $a,b \in A$ where $R_1^-(a) = \{a_1,\ldots,a_n\}$ and $R_1^-(b) = \{b_1,\ldots,b_{n'}\}$:

if $n \geq n'$ and there exists a permutation ρ over $R_1^-(a)$ such that $\forall i \in [1,n']$ $b_i \preceq a_{\rho(i)}$ and either $\exists j \in [1,n']$ such that $b_j \prec a_{\rho(j)}$ or $n > n'$

then $a \prec b$

If an argument b is at least as acceptable as an argument a according to **(CT)**, then if a has more attackers than b, or if there exists at least one attacker of a which is more acceptable than at least one attacker of b, then, according to **Strict Counter-Transitivity (SCT)** (also introduced in [1]), b is strictly more acceptable than a.

A given principle, when applied to a given graph, entails a preorder on its arguments.

Definition 7: Given a graph $\langle A,R \rangle$ with a and b elements of A, a principle α *entails* $a \preceq b$ iff $a \preceq b$ is true whenever α is satisfied.

Notice that every principle entails that an argument is at least as acceptable as itself.

Proposition 1: Given a graph $\langle A,R \rangle$ and $a \in A$, every principle entails $a \preceq a$.

Proof: Since $\preceq$ is reflexive, $a \preceq a$. Therefore, a principle entails $a \preceq a$ (in a degenerate way). ■

Example 4: Let us consider the graph of Figure 1.

- As b is the only argument which is not attacked, **(VP)** entails $\forall x \in A \setminus \{b\}$, $x \prec b$. In other words, b is strictly more acceptable than any other argument in this graph.
- **(CT)** entails $a \preceq c$, because a has an attacker, b, which is at least as acceptable as the attacker of c, which is also b (and, as we know, $b \preceq b$).
- **(SCT)** entails $a \prec c$, because $a \preceq c$ according to **(CT)**, and the number of attackers of a (two) is strictly greater than the number of attackers of c (one). Moreover, this principle also entails that $\forall x \in A \setminus \{b\}$, $x \prec b$, because b has no attacker, and hence, any other argument which has at least one attacker, has strictly more attackers than b, and these attackers are all at least as acceptable as the attackers of b (since there is none).

The last item suggests that, when **(SCT)** is used, **(VP)** is otiose, because what **(VP)** entails, can be entailed by **(SCT)**. Section V will investigate such *subsumption* cases.

To sum up, this is what is entailed by **(SCT)**:

$$a \prec c \prec b \quad \text{and} \quad d \prec b \quad \text{and} \quad e \prec b$$

This preorder is only partial: d and e are not compared, neither with each other, nor with a and c. A combination with other principles may however allow the entailment of a total preorder. Let us consider the following additional principle.

(CP) Given any $\langle A,R \rangle$, for all $a,b \in A$, if $|R_1^-(b)| < |R_1^-(a)|$ then $a \prec b$.

According to **Cardinality Precedence (CP)** (proposed in [1]), an argument whose number of attackers is lower than the number of attackers of another argument, is strictly more acceptable than this other argument.

Example 5: Given the graph of Figure 1, **(CP)** entails:

$$\begin{array}{lll} a \prec c \prec b & \text{and} & e \prec c \prec b \quad \text{and} \\ a \prec d \prec b & \text{and} & e \prec d \prec b \end{array}$$

It can be noticed that there is no contradiction between what is entailed by **(CP)** and what was entailed by the other principles. That is, when an argument was at least as acceptable as another one with one of these principles, **(CP)** does not entail that the latter is strictly more acceptable than the former. Such *incompatibility* cases will be investigated in Section VI.

Back to what **(CP)** entails, we can see that neither d and c are compared with each other, nor e and a. However, considering this preorder, **(SCT)** entails $c \prec d$ (because the attacker of c is strictly more acceptable than the attacker of e) and $a \prec e$ (because the attackers of a are more acceptable than the attackers of e). It can be noticed that what **(SCT)** entails given the preorder entailed by **(CP)**, is different from the one which was obtained without any input preorder (Example 4).

The preorder obtained by combining **(CP)** and **(SCT)** is:

$$a \quad \prec \quad e \quad \prec \quad c \quad \prec \quad d \quad \prec \quad b$$

Here, this combination of semantic principles entails the same (total) preorder as the one obtained with Dbs (Example 3).

However, it may not always be the case that a total preorder is entailed with this combination, nor that the entailed preorder is the same as the one given by Dbs. Actually, if one considers a graph where $A = \{a,b\}$ and $R = \{(a,b),(b,a)\}$, Dbs entails $a \simeq b$, whereas neither **(CP)** nor **(SCT)** (and nor their combination) entails any comparison between a and b.

V. Subsumption

When a user wants to combine several principles, it would be helpful to find out whether one of these principles is in fact not needed (because it follows from the other principles in the combination) —a few such results are given in [2], [14], [1].

Definition 8: The *subsumption of a principle P_1 by a principle P_2* is defined as follows: for all ranking-based semantics σ, if σ satisfies P_1 then σ satisfies P_2.

As Example 4 suggests, it has been shown in [1] that:

Proposition 2 ([1]): If a semantics σ satisfies **(SCT)** then σ satisfies **(VP)**.

Another case of subsumption involving **(VP)** is with **(CP)**.

Proposition 3: If a semantics σ satisfies **(CP)** then σ satisfies **(VP)**.

Proof: Let σ be a semantics satisfying **(CP)**. Clearly, the proviso for **(VP)**, i.e., $R_1^-(b) = \emptyset$ and $R_1^-(a) \neq \emptyset$, entails $|R_1^-(b)| < |R_1^-(a)|$ which is the proviso for **(CP)**. ■

An even more interesting subsumption case is shown next.

Proposition 4: If a semantics σ satisfies **(CT)** then σ satisfies **(EQ)**.

Proof: Let σ be a semantics satisfying **(CT)**. Assume the proviso for **(EQ)**, i.e., $|R_1^-(a)| = |R_1^-(b)|$ and for all $c \in R_1^-(a)$, $|\{x \in R_1^-(a) \mid x \simeq c\}| = |\{x \in R_1^-(b) \mid x \simeq c\}|$. We must show $a \simeq b$. For convenience, we write $R_1^-(a) = \{a_1,\dots,a_n\}$ and $R_1^-(b) = \{b_1,\dots,b_{n'}\}$. Trivially, $|R_1^-(a)| = |R_1^-(b)|$ gives $n \geq n'$. Second, the proviso $|\{x \in R_1^-(a) \mid x \simeq c\}| = |\{x \in R_1^-(b) \mid x \simeq c\}|$ shows that there is a bijection f between $R_1^-(a)$ and $R_1^-(b)$ such that $c \simeq f(c)$. Hence, there exists a permutation ρ over $R_1^-(a)$ such that $\forall i \in [1,n']\ b_i \preceq a_{\rho(i)}$. Since σ satisfies **(CT)**, $a \preceq b$ ensues. Clearly, $b \preceq a$ can be obtained similarly by symmetry between a and b in the proviso of **(EQ)**. ■

Of course, a principle can be subsumed by a group of principles none of which subsumes it alone.

(DP) Given any $\langle A,R\rangle$, for all $a,b \in A$, if $|R_1^-(a)| = |R_1^-(b)|$, $R_2^+(a) = \emptyset$ and $R_2^+(b) \neq \emptyset$ then $a \prec b$.

For two arguments with the same number of attackers, an argument which is defended is strictly more acceptable than an argument which is not; this is what is stated by the **Defence Precedence (DP)** (introduced in [1]) principle.

Proposition 5: If a semantics σ satisfies **(SCT)** and **(EQ)**, then σ also satisfies **(DP)**.

Proof: Let σ be a semantics satisfying **(SCT)** and **(EQ)**. We are to show that σ satisfies **(DP)**. Assume that

$$|R_1^-(a)| = |R_1^-(b)|, R_2^+(a) = \emptyset \text{ and } R_2^+(b) \neq \emptyset.$$

We must show $a \prec b$. Consider $a_i \in R_1^-(a)$. The assumption $R_2^+(a) = \emptyset$ implies $R_1^-(a_i) = \emptyset$. Consider $b_j \in R_1^-(b)$. There are two cases. (1) $R_1^-(b_j) = \emptyset$. Then, $|R_1^-(b_j)| = |R_1^-(a_i)|$. Moreover, a degenerate consequence of $R_1^-(b_j) = \emptyset$ is $\forall c \in R_1^-(b_j), |\{x \in R_1^-(b_j) \mid x \simeq c\}| = |\{x \in R_1^-(a_i) \mid x \simeq c\}|$ (because no such c exist). These are the proviso for **(EQ)** as applied to b_j and a_i, hence $b_j \simeq a_i$ ensues. Thus, $b_j \preceq a_i$. (2) $R_1^-(b_j) \neq \emptyset$. Then, $|R_1^-(a_i)| < |R_1^-(b_j)|$, so the first and third part of the proviso for **(SCT)** hold. Since $R_1^-(a_i) = \emptyset$, the second part of the proviso for **(SCT)** is satisfied also (because a forall on an empty set is true) and this gives $b_j \prec a_i$. Thus, $b_j \preceq a_i$.

Summing up, $b_j \preceq a_i$ for all $a_i \in R_1^-(a)$ and all $b_j \in R_1^-(b)$. However, $R_2^+(b) \neq \emptyset$ makes it that there exists $b_k \in R_1^-(b)$ s.t. $R_1^-(b_k) \neq \emptyset$. Also, $b_k \prec a_i$ as shown in case (2).

Therefore, the second part of the proviso for **(SCT)** holds (it suffices to take ρ to be identity). The assumption $|R_1^-(a)| = |R_1^-(b)|$ takes care of the first part of the proviso. Applying **(SCT)**, $a \prec b$ results. ■

VI. Incompatibility and floundering

Results on subsumption have shown that some principles are not needed when other principles are selected. It could be helpful as well to indicate to a user whether some principles are *incompatible*. This section defines what incompatibilities may be, and offers some results as to when such incompatibilities may arise.

In [2], the authors regard two axioms as incompatible if and only if there exist no semantics that can satisfy both axioms. This is of course well-taken but we prefer to avoid resorting to quantifying over all semantics. Instead, we adopt a sufficient (w.r.t. to [2]) condition taking into account the fact that a notion of contradiction in a logical setting may reveal an intricate matter (see [7] on the topic). Since principles have the form of if-then rules, a different standpoint makes sense as studied in [8].

A. Incompatibility of principles, weak form

*Definition 9 (**Incompatibility — weak**):* Two principles are *incompatible* iff there exists an $\langle A,R\rangle$ with some a and b in A such that one of these two principles entails $b \prec a$ and the other principle entails $a \preceq b$.

A simple example is with **(CP)** (introduced in Section V). In fact, **(CP)** is incompatible with **(SC)** in the sense of Definition 9. To verify, just take $A=\{a,b,c\}$ and aRa as well as aRb and cRb (see Figure 2(i)). Clearly, $R_1^-(a)=\{a\}$ and $R_1^-(b)=\{a,c\}$. Thus, $a\in R_1^-(a)$ while $b\notin R_1^-(b)$ hence **(SC)** gives $a\prec b$. However, $|R_1^-(a)|<|R_1^-(b)|$ holds that makes **(CP)** to give $b\prec a$.

As another illustration, **(CT)** and **(SC)** can be shown to be incompatible in the sense of Definition 9. There only needs to exhibit an argument graph that makes these two principles to conflict. Consider $A=\{a,b\}$ with aRa and aRb and $R_1^-(a)=\{a\}$ (see Figure 2(ii)). On the one hand, $a\in R_1^-(a)$ and $b\notin R_1^-(b)$ make **(SC)** to entail $a\prec b$. On the other hand, $R_1^-(a)=R_1^-(b)$ implies $n\geq n'$ wrt **(CT)** and also implies that there exists a permutation ρ over $R_1^-(a)$ such that $b_i\preceq a_{\rho(i)}$ for $i=1..|R_1^-(b)|$ (it is enough to take ρ to be identity). Hence, the proviso for **(CT)** is satisfied and $b\preceq a$ ensues. That is, **(CT)** entails $b\preceq a$.

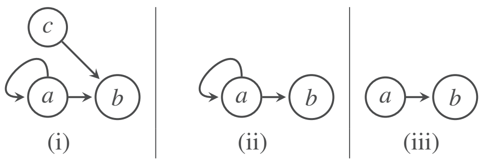

Fig. 2. Three argument graphs

Proposition 6: Incompatibility in the sense of Definition 9 is a symmetric relation.

Proposition 7: Let α be a principle "if $P_\alpha[a,b]$ then $a\prec b$" and β be a principle "if $P_\beta[a,b]$ then $a\prec b$" (one of these two occurrences of $\prec$ can be $\preceq$ instead) such that neither $\prec$ nor $\preceq$ occur in $P_\alpha[a,b]$ or $P_\beta[a,b]$. If $P_\alpha[a,b]\wedge P_\beta[b,a]$ is satisfiable then α and β are incompatible (in the sense of Definition 9).

Proof: Assume $P_\alpha[a,b]\wedge P_\beta[b,a]$ is satisfiable. Since neither $\prec$ nor $\preceq$ occur in $P_\alpha[a,b]$ or $P_\beta[a,b]$, there exists $\langle A,R\rangle$ such that $P_\alpha[a,b]\wedge P_\beta[b,a]$ holds for some a and b in A. Hence, the principle α entails $a\prec b$ and the principle β entails $b\prec a$. By Definition 9, α and β are incompatible (also in the case that one of these two occurrences of $\prec$ is $\preceq$ instead). ■

This notion of incompatibility is not limited to two principles, it can be extended to more principles. It only takes to check that the set of collectively incompatible principles can be partitioned into two subgroups, one subgroup that *together* entails $b\prec a$ and the other subgroup that *together* entails $a\preceq b$.

B. Floundering, weak version

A special case of incompatibility is when a principle is incompatible with itself.

*Definition 10 (**Floundering**):* α is a *floundering principle* iff there exists $\langle A,R\rangle$ and some $a\in A$ such that α entails $a\prec a$.

Proposition 8: α is a floundering principle iff α is incompatible (Definition 9) with itself.

Proof: If α is a floundering principle then there exists an $\langle A,R\rangle$ such that $a\prec a$ for some $a\in A$. Moreover, $a\preceq a$ since $\preceq$ is reflexive. Definition 9 is thus satisfied for the case $\beta=\alpha$ by taking $b=a$. Conversely, assume that α is incompatible with itself. There must exist some $\langle A,R\rangle$ such that, for some $a,b\in A$, α entails $b\prec a$ and α entails $a\preceq b$. Since $\preceq$ is transitive, $a\prec a$ ensues. Then, α is a floundering principle. ■

There exist principles, albeit intuitively self-contradictory, that are not floundering. (Floundering principles —that is, following Proposition 8, principles that are incompatible with themselves— do not exhaust all cases of intuitively self-contradictory principles.) An example is

$$\text{If } b\prec a \text{ then } a\preceq b.$$

This weird principle is such that, if $b\prec a$ for some b in the graph, then $a\prec a$ (for all a in the graph) by transitivity. Yet, $a\prec a$ is not entailed because the proviso $b\prec a$ need not hold (in isolation, this principle only induces the identity pre-order).

Proposition 9: If α is a floundering principle then α is incompatible (Definition 9) with every principle β.

C. Incompatibility of principles, strong form

Proposition 7 seems to indicate that the notion of incompatibility arising from Definition 9 is too weak in the sense that it fails to capture systematic contradiction, capturing only contradiction within some graphs.

Definition 11: A principle α *opposes* a principle β iff for all $\langle A,R\rangle$, all a,b in A, if $P_\alpha[a,b]$ holds then α entails $P_\beta[b,a]$.

Proposition 10: Let α oppose β. For all $\langle A,R\rangle$ and all a,b in A such that $P_\alpha[a,b]$ holds, if both α and β are satisfied then $a\simeq b$.

Proof: Let $\langle A,R\rangle$ and $a,b\in A$ with $P_\alpha[a,b]$. Assume both α and β are satisfied. Then, $a\preceq b$. However, $P_\beta[b,a]$ is true because α opposes β. Since β is satisfied, $b\preceq a$. ■

Writing $b_1,\ldots,b_n$ for all b's such that $P_\alpha[a,b]$, Proposition 10 thus means that if α opposes β then $a\simeq b_1\simeq\ldots\simeq b_n$ (when both α and β are satisfied).

Proposition 10 expresses that two opposing principles give conclusions with converse ordering but no conflict need to arise. To go from opposing principles to incompatible principles, the notion of contradiction is to be explicited: in the sequel, contradictory in fact means contradictory in the context of preorders.

*Definition 12 (**Incompatibility — strong**):* Two principles α and β are *strongly incompatible* iff $P_\alpha[a,b] \wedge P_\beta[b,a] \wedge C_\alpha[a,b] \wedge C_\beta[b,a]$ is contradictory and either α opposes β or β opposes α.

Proposition 11: Let α and β be two principles such that α or β is of the form "if ... then $a \prec b$". If α opposes β then α and β are strongly incompatible.

Proof: Should $C_\alpha[a,b]$ be $a \prec b$, $C_\alpha[a,b] \wedge C_\beta[b,a]$ implies $a \prec b \wedge b \preceq a$. Should $C_\beta[a,b]$ be $a \prec b$, $C_\alpha[a,b] \wedge C_\beta[b,a]$ implies $a \preceq b \wedge b \prec a$. In both cases, $P_\alpha[a,b] \wedge P_\beta[b,a] \wedge C_\alpha[a,b] \wedge C_\beta[b,a]$ is contradictory. ■

Except for the cases where both α and β have a conclusion $a \preceq b$, Proposition 11 can be taken as a (much) simpler definition for strong incompatibility.

Proposition 12: Strong incompatibility (Definition 12) is a symmetric relation.

The reason for the stronger requirement in Definition 12 is that a unique pathological graph should not be enough to make two principles incompatible as is the case with Definition 9 or the notion in [2].

It can be observed that the graph $A = \{a,b\}$ with aRa and aRb (Figure 2(ii)) used to show that **(CT)** and **(SC)** are incompatible in the sense of Definition 9 fails to ensure that arbitrary graphs allow the proviso for **(SC)** to be entailed by the conclusion of **(CT)**. Indeed, consider the graph $A = \{a,b\}$ with aRb (Figure 2(iii)). Since $P_{SC}[b,a]$ is about the structure of the graph but fails in it, $P_{SC}[b,a]$ cannot be obtained from $P_{CT}[a,b]$. The other way around, $P_{CT}[b,a]$ is $|R_1^-(b)| \geq |R_1^-(a)|$ which fails to follow from $P_{SC}[a,b]$, namely aRa and $b \not R b$.

On a more general level, the notion in [2] or Definition 9 makes any quantity-based property to be incompatible with any quality-based property despite the fact that they are graphs that allow these to underlie similar conclusions (granted, it will not be the case in most graphs).

Here is an illustration of two principles incompatible according to Definition 12. On the one hand, the following principle (Def) roughly expresses that extra defenders cannot make an argument less acceptable:

$$\text{if } R_2^+(a) \subseteq R_2^+(b) \text{ then } a \preceq b.$$

On the other hand, the next principle (Con) —it is a consequence of **(CP)**— means that the less counter-arguments (regardless of whether they are defended against) an argument has, the more acceptable it is:

$$\text{if } R_1^-(b) \subset R_1^-(a) \text{ then } a \prec b.$$

Structurally, if $R_1^-(x) \subset R_1^-(y)$ then $R_2^+(x) \subseteq R_2^+(y)$. Therefore, if $P_{Con}[a,b]$ then $P_{Def}[b,a]$. It is enough to apply Definition 11 and Proposition 11.

D. Floundering, strong version

*Definition 13 (**Floundering — Strong**):* α is a *strongly floundering principle* iff for all $\langle A,R\rangle$ and all $a \in A$, if $P_\alpha[a,b]$ holds for some $b \in A$ then α entails $a \prec a$.

Proposition 13: α is a strongly floundering principle iff α is strongly incompatible with itself.

Proof: ($\longleftarrow$) Let α be self-incompatible, i.e., α opposes itself. Consider $\langle A,R\rangle$ and $a,b \in A$ such that $P_\alpha[a,b]$ holds. Assume $a \prec a$ does not hold. Since $P_\alpha[a,b]$ holds, α entails $P_\alpha[a,b]$. As α opposes itself, α entails $P_\alpha[b,a]$, too. In turn, α entails $C_\alpha[b,a]$. To sum up, α entails $P_\alpha[a,b] \wedge P_\alpha[b,a] \wedge C_\alpha[a,b] \wedge C_\alpha[b,a]$. Definition 12 means that $P_\alpha[a,b] \wedge P_\alpha[b,a] \wedge C_\alpha[a,b] \wedge C_\alpha[b,a]$ is contradictory. Hence, α entails $a \prec a$. ($\longrightarrow$) Let α be a strongly floundering principle. Let $\langle A,R\rangle$ and $a,b \in A$ such that $P_\alpha[a,b]$ holds. So, α entails $a \prec a$. That α is satisfied makes only $C_\alpha[a,b]$ to ensue in the presence of $P_\alpha[a,b]$. That is, $a \prec a$ is a consequence of $P_\alpha[a,b] \wedge C_\alpha[a,b]$. Then, $P_\alpha[a,b] \wedge P_\alpha[b,a] \wedge C_\alpha[a,b] \wedge C_\alpha[b,a]$ is contradictory. ■

The dubious principle given in Section VI-B, namely

$$\text{If } b \prec a \text{ then } a \preceq b$$

is a strongly floundering principle in the sense of Definition 13.

VII. Conclusion

This paper discusses incompatibilities that may arise when combining principles for the specification of ranking-based argumentation semantics. It also provides results about subsumption between such principles. Other similar results between principles may be studied in the future, and other principles may be defined. Studying an equivalence between some combinations of semantic principles, and existing ranking-based semantics, is also an interesting research avenue.

The results given here are a step towards a generalization of the construction of ranking-based semantics. Such an approach may be, in the future, encoded in logic, and implemented in the SESAME software. The software would then provide a logical encoding of the semantics the user would specify as a combination of principles. When applied to a given argument graph, the models of the instantiated formula would correspond to the rankings according to the specified semantics. It would then be possible to compute the rankings by feeding the instantiated formula to a SAT solver. An effective path from semantics definition to semantics computation would then be drawn.

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